

EXAMPLE 6 Let $f(x) = \sqrt{x+5} - \sqrt{x-7}$. Find all x -values for which $f(x) = 2$.

SOLUTION We must have $f(x) = 2$, or

$$\sqrt{x+5} - \sqrt{x-7} = 2. \quad \text{Substituting for } f(x)$$

To solve, we isolate one radical term and square both sides:

$$\sqrt{x+5} = 2 + \sqrt{x-7}$$

$$(\sqrt{x+5})^2 = (2 + \sqrt{x-7})^2$$

$$x + 5 = 4 + 4\sqrt{x-7} + (x-7)$$

$$5 = 4\sqrt{x-7} - 3$$

$$8 = 4\sqrt{x-7}$$

$$2 = \sqrt{x-7}$$

$$2^2 = (\sqrt{x-7})^2$$

$$4 = x - 7$$

$$11 = x.$$

Check: $f(11) = \sqrt{11+5} - \sqrt{11-7}$
 $= \sqrt{16} - \sqrt{4}$
 $= 4 - 2 = 2.$

We have $f(x) = 2$ when $x = 11$.

■ Try Exercise 49.

Isolate a radical term.

Raise both sides to the same power.

Isolate a radical term.

Raise both sides to the same power.

Solve.

Check.

Adding $\sqrt{x-7}$ to both sides. This isolates one of the radical terms.

Using the principle of powers (squaring both sides)

Using $(A+B)^2 = A^2 + 2AB + B^2$

Adding $-x$ to both sides and combining like terms

Isolating the remaining radical term

Squaring both sides

7.6

Exercise Set

FOR EXTRA HELP



Concept Reinforcement Classify each of the following statements as either true or false.

1. If $x^2 = 25$, then $x = 5$. **False**
2. If $t = 7$, then $t^2 = 49$. **True**
3. If $\sqrt{x} = 3$, then $(\sqrt{x})^2 = 3^2$. **True**
4. If $x^2 = 36$, then $x = 6$. **False**
5. $\sqrt{x} - 8 = 7$ is equivalent to $\sqrt{x} = 15$. **True**
6. $\sqrt{t} + 5 = 8$ is equivalent to $\sqrt{t} = 3$. **True**

Solve.

- | | |
|-------------------------------------|-------------------------------------|
| 7. $\sqrt{5x+1} = 4$ 3 | 8. $\sqrt{7x-3} = 5$ 4 |
| 9. $\sqrt{3x+1} = 6$ $\frac{25}{3}$ | 10. $\sqrt{2x-1} = 2$ $\frac{9}{2}$ |
| 11. $\sqrt{y+1} - 5 = 8$ 168 | 12. $\sqrt{x-2} - 7 = -4$ 11 |
| 13. $\sqrt{8-x} + 7 = 10$ -1 | 14. $\sqrt{y+4} + 6 = 7$ -3 |
| 15. $\sqrt[3]{x+5} = 2$ 3 | 16. $\sqrt[3]{x-2} = 3$ 29 |
| 17. $\sqrt[4]{y-1} = 3$ 82 | 18. $\sqrt[4]{x+3} = 2$ 13 |
| 19. $3\sqrt{x} = x$ 0, 9 | 20. $8\sqrt{y} = y$ 0, 64 |

21. $2y^{1/2} - 13 = 7$ 100

22. $3x^{1/2} + 12 = 9$

23. $\sqrt[3]{x} = -3$ -27

24. $\sqrt[3]{y} = -4$ -64

25. $z^{1/4} + 8 = 10$ 16

26. $x^{1/4} - 2 = 1$ 81

Aha! 27. $\sqrt{n} = -2$ No solution

28. $\sqrt{a} = -1$ No solution

29. $\sqrt[4]{3x+1} - 4 = -1$ $\frac{80}{3}$

30. $\sqrt[4]{2x+3} - 5 = -2$ 39

31. $(21x + 55)^{1/3} = 10$ 45

32. $(5y + 31)^{1/4} = 2$ -3

33. $\sqrt[3]{3y+6} + 7 = 8$ $-\frac{5}{3}$

34. $\sqrt[3]{6x+9} + 5 = 2$ -6

35. $\sqrt{3t+4} = \sqrt{4t+3}$ 1

36. $\sqrt{2t-7} = \sqrt{3t-12}$ 5

37. $3(4-t)^{1/4} = 6^{1/4}$ $\frac{106}{27}$

38. $2(1-x)^{1/3} = 4^{1/3}$ $\frac{1}{2}$

39. $3 + \sqrt{5-x} = x$ 4

40. $x = \sqrt{x-1} + 3$ 5

41. $\sqrt{4x-3} = 2 + \sqrt{2x-5}$ 3, 7

42. $3 + \sqrt{z-6} = \sqrt{z+9}$ 7

43. $\sqrt{20-x} + 8 = \sqrt{9-x} + 11$ $\frac{80}{9}$

44. $4 + \sqrt{10-x} = 6 + \sqrt{4-x}$ $\frac{15}{4}$

45. $\sqrt{x+2} + \sqrt{3x+4} = 2$ -1

46. $\sqrt{6x+7} - \sqrt{3x+3} = 1$ $-1, \frac{1}{3}$

47. If $f(x) = \sqrt{x} + \sqrt{x-9}$, find any x for which $f(x) = 1$. No solution

48. If $g(x) = \sqrt{x} + \sqrt{x-5}$, find any x for which $g(x) = 5$. 9

49. If $f(t) = \sqrt{t-2} - \sqrt{4t+1}$, find any t for which $f(t) = -3$. 2, 6

50. If $g(t) = \sqrt{2t+7} - \sqrt{t+15}$, find any t for which $g(t) = -1$. 1

51. If $f(x) = \sqrt{2x-3}$ and $g(x) = \sqrt{x+7} - 2$, find any x for which $f(x) = g(x)$. 2

52. If $f(x) = 2\sqrt{3x+6}$ and $g(x) = 5 + \sqrt{4x+9}$, find any x for which $f(x) = g(x)$. 10

53. If $f(t) = 4 - \sqrt{t-3}$ and $g(t) = (t+5)^{1/2}$, find any t for which $f(t) = g(t)$. 4

54. If $f(t) = 7 + \sqrt{2t-5}$ and $g(t) = 3(t+1)^{1/2}$, find any t for which $f(t) = g(t)$. 15

TW 55. Explain in your own words why it is important to check your answers when using the principle of powers.

TW 56. The principle of powers is an “if-then” statement that becomes false when the sentence parts are interchanged. Give an example of another such if-then statement from everyday life (answers will vary).

SKILL REVIEW

To prepare for Section 7.7, review finding dimensions of triangles and rectangles (Sections 1.7 and 5.8).

Solve.

57. **Sign Dimensions.** The largest sign in the United States is a rectangle with a perimeter of 430 ft. The length of the rectangle is 5 ft longer than thirteen times the width. Find the dimensions of the sign. [1.7]

Source: Florida Center for Instructional Technology

Length: 200 ft; width: 15 ft

58. **Sign Dimensions.** The base of a triangular sign is 4 in. longer than twice the height. The area of the sign is 255 in^2 . Find the dimensions of the sign. [5.8]
Base: 34 in.; height: 15 in.

59. **Photograph Dimensions.** A rectangular family photo is 4 in. longer than it is wide. The area of the photo is 140 in^2 . Find the dimensions of the photograph. [5.8]

Length: 14 in.; width: 10 in.

60. **Sidewalk Length.** The length of a rectangular lawn between classroom buildings is 2 yd less than twice the width of the lawn. A path that is 34 yd long stretches diagonally across the area. What are the dimensions of the lawn? [5.8] Length: 30 yd; width: 16 yd

61. The sides of a right triangle are consecutive even integers. Find the length of each side. [5.8] 6, 8, 10

62. One leg of a right triangle is 5 cm long. The hypotenuse is 1 cm longer than the other leg. Find the length of the hypotenuse. [5.8] 13 cm

SYNTHESIS

TW 63. Describe a procedure that could be used to create radical equations that have no solution.

TW 64. Is checking essential when the principle of powers is used with an odd power n ? Why or why not?

65. **Firefighting.** The velocity of water flow, in feet per second, from a nozzle is given by

$$v(p) = 12.1\sqrt{p},$$

where p is the nozzle pressure, in pounds per square inch (psi). Find the nozzle pressure if the water flow is 100 feet per second. About 68 psi

Source: Houston Fire Department Continuing Education

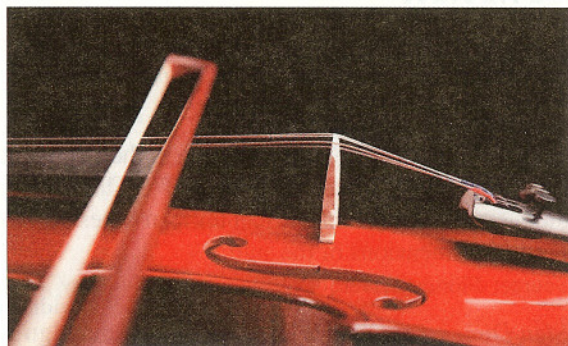
66. **Firefighting.** The velocity of water flow, in feet per second, from a water tank that is h feet high is given by

$$v(h) = 8\sqrt{h}.$$

Find the height of a water tank that provides a water flow of 60 feet per second. 56.25 ft

Source: Houston Fire Department Continuing Education

67. **Music.** The frequency of a violin string varies directly with the square root of the tension on the string. A violin string vibrates with a frequency of 260 Hz when the tension on the string is 28 N. What is the frequency when the tension is 32 N? **About 278 Hz**



68. **Music.** The frequency of a violin string varies inversely with the square root of the density of the string. A nylon violin string with a density of 1200 kg/m^3 vibrates with a frequency of 250 Hz. What is the frequency of a silk violin string with a density of 1300 kg/m^3 ? **About 240 Hz**
Source: www.speech.kth.se

Steel Manufacturing. In the production of steel and other metals, the temperature of the molten metal is so great that conventional thermometers melt. Instead, sound is transmitted across the surface of the metal to a receiver on the far side and the speed of the sound is measured. The formula

$$S(t) = 1087.7 \sqrt{\frac{9t + 2617}{2457}}$$

gives the speed of sound $S(t)$, in feet per second, at a temperature of t degrees Celsius.



69. Find the temperature of a blast furnace where sound travels 1880 ft/sec. **524.8°C**

70. Find the temperature of a blast furnace where sound travels 1502.3 ft/sec. **230°C**

71. Solve the above equation for t . \square

Automotive Repair. For an engine with a displacement of 2.8 L, the function given by

$$d(n) = 0.75\sqrt{2.8n}$$

can be used to determine the diameter size of the carburetor's opening, in millimeters. Here n is the number of rpm's at which the engine achieves peak performance.

Source: macdizzy.com

About 4166 rpm

72. If a carburetor's opening is 81 mm, for what number of rpm's will the engine produce peak power?
73. If a carburetor's opening is 84 mm, for what number of rpm's will the engine produce peak power? **4480 rpm**

Escape Velocity. A formula for the escape velocity v of a satellite is

$$v = \sqrt{2gr} \sqrt{\frac{h}{r+h}}$$

where g is the force of gravity, r is the planet or star's radius, and h is the height of the satellite above the planet or star's surface.

74. Solve for h . \square

75. Solve for r . \square

Solve.

76. $\left(\frac{z}{4} - 5\right)^{2/3} = \frac{1}{25} \frac{2504}{125}, \frac{2496}{125}$

77. $\frac{x + \sqrt{x+1}}{x - \sqrt{x+1}} = \frac{5}{11} \quad -\frac{8}{9}$

78. $\sqrt{\sqrt{y} + 49} = 7 \quad 0$

79. $(z^2 + 17)^{3/4} = 27 \quad -8, 8$

80. $x^2 - 5x - \sqrt{x^2 - 5x - 2} = 4$
(Hint: Let $u = x^2 - 5x - 2$.) $-1, 6$

81. $\sqrt{8-b} = b\sqrt{8-b} \quad 1, 8$

Without graphing, determine the x -intercepts of the graphs given by each of the following.

82. $f(x) = \sqrt{x-2} - \sqrt{x+2} + 2 \quad (2, 0)$

83. $g(x) = 6x^{1/2} + 6x^{-1/2} - 37 \quad \left(\frac{1}{36}, 0\right), (36, 0)$

84. $f(x) = (x^2 + 30x)^{1/2} - x - (5x)^{1/2} \quad (0, 0), \left(\frac{125}{4}, 0\right)$

Try Exercise Answers: Section 7.6

7. 3 27. No solution 33. $-\frac{5}{3}$ 39. 4 41. 3, 7 49. 2, 6